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Scattering of Sound Waves in a Lined Two-stepped Waveguide

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Abstract

Scattering of sound waves in a lined two-stepped cylindrical duct is investigated by using the Wiener-Hopf technique. The problem is reduced to a pair of modified Wiener-Hopf equations by using the Fourier transform for the scattered field and applying the boundary conditions in the transform domain. The solution consists of four sets of infinitely many constants which provide four systems of linear algebraic equations.

Key words: Wiener-Hopf , Fourier Transform, Step discontinues , Absorbent lining

1. Introduction

Acoustic waveguides are effective tools for transferring of acoustic energy. Unfortunately, some unwanted voices are produced during this energy transfer through the waveguide. These unwanted voices are a major problem for industry and therefore for the quality of life of people. So, it has been a field of study by researchers for many years. The presence of sudden area changes in the waveguide or the coating of the waveguide with internal sound absorbing material can be used to reduce unwanted voice. In this context, sudden area changes in the waveguide and internal coating of the waveguide have become a focus for scientists.

Wiener-Hopf Technique is the efficient method for investigation of scattering and radiation problems from the ducts. Rawlins considered the radiation of sound from an unflanged rigid cylindrical duct with an acoustically absorbing internal surface by using the Wiener-Hopf Technique [1]. The modes of an infinite duct, paying particular attention to possible instabilities are analyzed by Nilsson and Brander [2] and they also investigated the reflection and transmission of sound in a cylindrical waveguide with a jump in its diameter is treated through the Wiener-Hopf Technique [3]. The problem of propagation of sound in an infinite rigid circular cylindrical duct with an inserted expansion chamber whose walls are treated with an acoustically absorbent material is treated rigously through the Wiener-Hopf Technique is solved by Buyukaksoy and Demir [4].

In this study, a two-stepped cylindrical waveguide whose walls are uniformly covered with an acoustically absorbent material is considered. The problem is divided into zones according to the model and the necessary boundary and continuity conditions are determined. Then, the problem is reduced to a pair of modified Wiener-Hopf equations by using the Fourier transform for the scattered field and applying the boundary conditions in the transform domain. The solution consists of four sets of infinitely many unknown expansion coefficients providing four systems of linear algebraic equations. The time dependence is assumed to be $exp(-i\omega t)$ with ω being the

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angular frequency and suppressed throughout this paper.

2. Analysis

2.1.Formulation of the Problem

The geometry of this problem consists of an infinite cylindrical duct with two area expansions at z=0 and z=l. Duct walls are assumed to be infinitely thin and they defined by $\{\rho = a, z \in (-\infty, 0)\} \cup \{\rho \in (a, b), z = 0\} \cup \{\rho = b, z \in (0, l)\} \cup \{\rho \in (b, c), z = l\} \cup \{\rho = c, z \in (l, \infty)\}$ where (ρ, θ, z) denote the cylindrical polar coordinates. Also it is assumed that inner surface of duct is treated by an acoustically absorbent lining which is denoted by η (see Fig. 1).

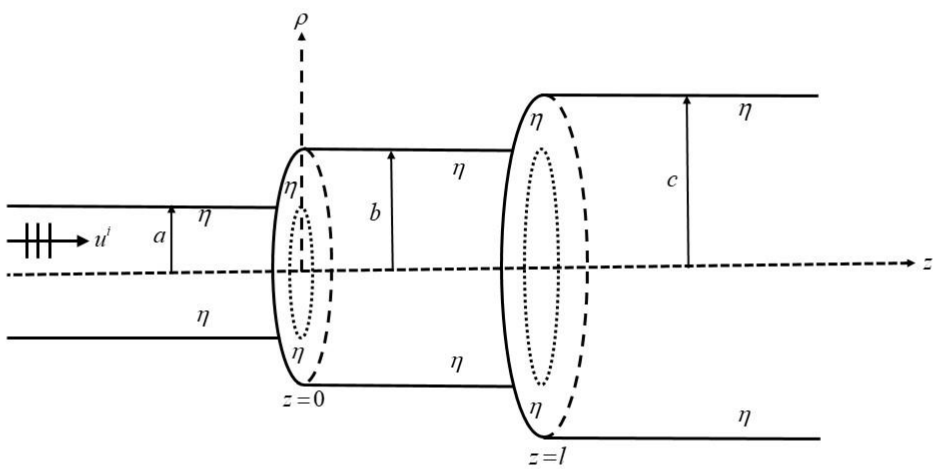

Figure 1. Geometry of the problem

From the symmetry of the geometry of the problem and of the incident field, the total field everywhere will be independent of θ. The incident sound wave propagating along the positive z direction and is defined by

$$u^i(\rho, z) = A_n J_0(\gamma_n \rho / a) e^{i\lambda_n z} \quad , n = 1,2, \dots \tag{1}$$

$$ika\eta J_0(\gamma_n) + \gamma_n J_1(\gamma_n) = 0 \tag{2}$$

$$\lambda_n = \sqrt{k^2 - (\gamma_n / a)^2} \tag{3}$$

Here $k = \omega / c$ denotes the wave number of the space and c is the speed of the sound. A_n stands for the amplitude of the incident wave. For the sake of analytical convenience, the total field can be written in different regions as:

$$u^T(\rho,z)=\begin{cases}u_1(\rho,z)+u^i(\rho,z) & ; \quad \rho<a\,, \quad -\infty<z<0\\ u_2(\rho,z) & ; \quad \rho\in(a,b), \quad 0<z<\infty\\ u_3(\rho,z) & ; \quad \rho\in(b,c), \quad l<z<\infty\end{cases} \tag{4}$$

where u^i is the incident field as given by (1) and the fields $u_j(\rho,z), j=1,2,3$ which satisfy the Helmholtz equation,

$$\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right)+\frac{\partial^2}{\partial z^2}+k^2\right]u_j(\rho,z)=0 \quad , \quad j=1,2,3 \tag{5}$$

is to be determined with the help of the following boundary and continuity relations:

$$\left(ik\eta-\frac{\partial}{\partial\rho}\right)u_1(a,z)=0 \quad , \quad z<0 \quad , \qquad \left(ik\eta-\frac{\partial}{\partial\rho}\right)u_2(b,z)=0 \;\; , \;\; z>0 \qquad (6,7)$$

$$\left(ik\eta+\frac{\partial}{\partial z}\right)u_2(\rho,0)=0 \;\; , \;\; a<\rho<b \quad , \quad \left(ik\eta-\frac{\partial}{\partial\rho}\right)u_3(c,z)=0 \quad , \quad z>l \qquad (8,9)$$

$$\left(ik\eta+\frac{\partial}{\partial z}\right)u_3(\rho,l)=0 \;\; , \;\; b<\rho<c \quad , \quad u_1(a,z)+u^i(a,z)=u_2(a,z) \qquad (10,11)$$

$$\frac{\partial}{\partial\rho}\left[u_1(a,z)+u^i(a,z)\right]=\frac{\partial}{\partial\rho}u_2(a,z)\,,u_2(b,z)=u_3(b,z)\,,\frac{\partial}{\partial\rho}u_2(b,z)=\frac{\partial}{\partial\rho}u_3(b,z) \quad (12-14)$$

2.2.Derivation of the Modified Wiener-Hopf Equations

Fourier transform of the Helmholtz equation in the region $\rho<a$ for $\boldsymbol{z}\in(-\infty,\infty)$ is

$$\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right)+(k^2-\alpha^2)\right]F(\rho,\alpha)=0 \quad . \tag{15}$$

Here $F(\rho,\alpha)$ is the Fourier transform of the field $u_1(\rho,z)$ defined to be

$$F(\rho,\alpha)=\int_{-\infty}^{\infty}u_1(\rho,z)e^{i\alpha z}dz=F_-(\rho,\alpha)+F_+(\rho,\alpha) \tag{16}$$

$F_{-}(\rho,\alpha)$ ve $F_{+}(\rho,\alpha)$ are half-plane analytical functions on complex$-\alpha$ plane (see Fig. 2) defined by fourier integrals as:

$$F^{-}(\rho,\alpha) = \int_{-\infty}^{0} u_1(\rho,z)e^{i\alpha z}dz \quad , \quad F_{+}(\rho,\alpha) = \int_{0}^{\infty} u_1(\rho,z)e^{i\alpha z}dz \tag{17,18}$$

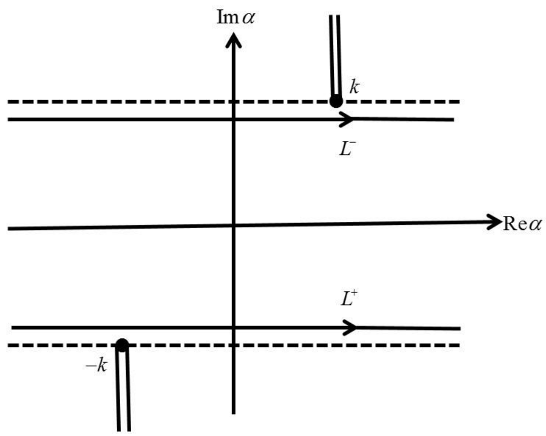

Figure 2. Complex$-\alpha$ plane

The solution of (15) due to the analytical properties of $F_{\pm}(\rho,\alpha)$ is as follows

$$F_{-}(\rho,\alpha) + F_{+}(\rho,\alpha) = \frac{J_0(K\rho)}{J(a,\alpha)}\Phi_1^{+}(a,\alpha) \tag{19}$$

Here $K(\alpha) = \sqrt{k^2-\alpha^2}$ is the square root function defined in complex$-\boldsymbol{\alpha}$ plane.

$$\Phi_1^{+}(a,\alpha) = ik\eta F_{+}(a,\alpha) - \dot{F}_{+}(a,\alpha) \tag{20}$$

and

$$J(a,\alpha) = ik\eta J_0(Ka) + KJ_1(Ka) \tag{21}$$

J_n is the Bessel function of integer order. Consider the $\rho \in (a,b)$ and $z \in (0,\infty)$ regions in which the scattered field $u_2(\rho,z)$ provides the Helmholtz equation. The Fourier transform of this equation in this region is

$$\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right) + K^2(\alpha)\right]\left\{G_1(\rho,\alpha) + e^{i\alpha l}G_{+}(\rho,\alpha)\right\} = -(ik\eta_2 + i\alpha)f(\rho) \tag{22}$$

where

$$f(\rho) = u_2(\rho,0)\ . \tag{23}$$

$G_1(\rho,\alpha)$ is an entire function and $G_+(\rho,\alpha)$ is a regular function in the upper half of the complex-α plane which are defined as, respectively

$$G_1(\rho,\alpha)=\int_0^l u_2(\rho,z)e^{i\alpha z}dz \quad , \quad G_+(\rho,\alpha)=\int_l^\infty u_2(\rho,z)e^{i\alpha(z-l)}dz \qquad (24a,b)$$

The general solution of (22) is obtained with the help of Green's function tecnique as follows, without going into details

$$\begin{aligned}G_1(\rho,\alpha)+e^{i\alpha l}G_+(\rho,\alpha)\\ =\frac{1}{L(\alpha)}\{\{[ik\eta G_1(a,\alpha)-\dot{G}_1(a,\alpha)]+e^{i\alpha l}[ik\eta G_+(a,\alpha)-\dot{G}_+(a,\alpha)]\\ \times[Y(b,\alpha)J_0(K\rho)-J(b,\alpha)Y_0(K\rho)]\\ -\{[ik\eta G_1(a,\alpha)-\dot{G}_1(a,\alpha)]+e^{i\alpha l}[ik\eta G_+(a,\alpha)-\dot{G}_+(a,\alpha)]\}\\ \times[Y(a,\alpha)J_0(K\rho)-J(a,\alpha)Y_0(K\rho)]-\int_a^b(ik\eta+i\alpha)f(t)Q_1(\rho,t,\alpha)tdt\}\end{aligned} \qquad (25)$$

where

$$Q_1(\rho,t,\alpha)=\frac{\pi}{2}\begin{cases}[J(a,\alpha)Y_0(K\rho)-Y(a,\alpha)J_0(K\rho)]\times[J(b,\alpha)Y_0(Kt)-Y(b,\alpha)J_0(Kt)] & ,\rho<t\\ [J(b,\alpha)Y_0(K\rho)-Y(b,\alpha)J_0(K\rho)]\times[J(a,\alpha)Y_0(Kt)-Y(a,\alpha)J_0(Kt)] & ,\rho>t\end{cases} \qquad (26)$$

$$L(\alpha)=Y(b,\alpha)J(a,\alpha)-J(b,\alpha)Y(a,\alpha) \quad , \qquad (27a)$$

$$Z(a,\alpha)=ik\eta Z_0(Ka)+KZ_1(Ka) \;, \quad Z(b,\alpha)=ik\eta Z_0(Kb)+KZ_1(Kb) \qquad (27b,c)$$

$Z=J,Y$ are Bessel fonctions. Using continuity relation in (11) and (12) together with (25) and taking into account (19) give

$$\begin{aligned}F_-(a,\alpha)+\frac{2e^{i\alpha l}}{\pi aL(\alpha)}\Phi_2^+(b,\alpha)-\frac{2}{\pi a}\frac{J(b,\alpha)}{J(a,\alpha)L(\alpha)}\Phi_1^+(a,\alpha)\\ =\frac{1}{aL(\alpha)}\int_a^b(ik\eta+i\alpha)f(t)[J(b,\alpha)Y_0(Kt)-Y(b,\alpha)J_0(Kt)]tdt-\frac{A_nJ_0(\gamma_n)}{i(\lambda_n+\alpha)}\end{aligned} \qquad (28)$$

where

$$\Phi_2^+(b,\alpha) = \left[ik\eta G_+(a,\alpha) - \dot{G}_+(a,\alpha)\right] \tag{29}$$

Making necessary arrangements, we obtain fisrt Modified Wiener-Hopf equation valid in the strip $Im(-k) < Im\alpha < Im(k)$,

$$\frac{a}{2}F_-(a,\alpha) - V_1(\alpha)\Phi_1^+(a,\alpha) + \frac{e^{i\alpha l}}{\pi L(\alpha)}\Phi_2^+(b,\alpha) = \sum_{m=1}^{\infty}\frac{J(b,\alpha_m)}{\pi J(a,\alpha_m)}\frac{(ik\eta + i\alpha)f_m}{\alpha_m^2 - \alpha^2} - \frac{a}{2}\frac{A_n J_0(\gamma_n)}{i(\lambda_n + \alpha)} \tag{30}$$

here

$$V_1(\alpha) = \frac{J(b,\alpha)}{\pi J(a,\alpha)L(\alpha)} \tag{31}$$

Now consider the $\rho \in (b,c)$ and $z \in (l,\infty)$ regions in which the scattered field $u_3(\rho,z)$ provides the Helmholtz equation. The Fourier transform of this equation in this region is

$$\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right) + K^2(\alpha)\right]H_+(\rho,\alpha) = -(ik\eta + i\alpha)g(\rho) \tag{32}$$

where

$$g(\rho) = u_3(\rho,l)\ . \tag{33}$$

$H_+(\rho,\alpha)$ is a regular function in the upper half of the complex-α plane which are defined as

$$H_+(\rho,\alpha) = \int_l^{\infty} u_3(\rho,z)e^{i\alpha(z-l)}dz \tag{34}$$

The general solution of (32) is obtained with the help of Green's function tecnique as follows, without going into details

$$H_+(\rho,\alpha) = \frac{1}{N(\alpha)}\{[ik\eta H_+(b,\alpha) - \dot{H}_+(b,\alpha)][Y(c,\alpha)J_0(K\rho) - J(c,\alpha)Y_0(K\rho)] = -\int_b^c (ik\eta + i\alpha)g(t)Q_2(\rho,t,\alpha)tdt\} \tag{35}$$

where

$$Q_2(\rho,t,\alpha) = \frac{\pi}{2}\begin{cases}[J(b,\alpha)Y_0(K\rho) - Y(b,\alpha)J_0(K\rho)] \times [J(c,\alpha)Y_0(Kt) - Y(c,\alpha)J_0(Kt)] & ,\rho < t \\ [J(c,\alpha)Y_0(K\rho) - Y(c,\alpha)J_0(K\rho)] \times [J(b,\alpha)Y_0(Kt) - Y(b,\alpha)J_0(Kt)] & ,\rho > t\end{cases} \tag{36}$$

$$N(\alpha) = Y(c,\alpha)J(b,\alpha) - J(c,\alpha)Y(b,\alpha), \quad Z(c,\alpha) = ik\eta Z_0(Kc) + KZ_1(Kc) \tag{37a,b}$$

$Z = J, Y$ are Bessel fonctions. Using continuity relation in (13) and (14) together with (25) and taking into account (25) and making necessary arrengements give

$$-\frac{1}{L(\alpha)}\Phi_1^+(a,\alpha) + \frac{M(\alpha)e^{i\alpha l}}{L(\alpha)N(\alpha)}\Phi_2^+(b,\alpha) - \frac{\pi b}{2}G_1(b,\alpha)$$

$$= -e^{i\alpha l}\sum_{m=1}^{\infty}\frac{J(c,\beta_m)}{J(b,\beta_m)}\frac{(ik\eta + i\alpha)g_m}{\beta_m^2 - \alpha^2} + \sum_{m=1}^{\infty}\frac{(ik\eta + i\alpha)f_m}{\alpha_m^2 - \alpha^2} \tag{38}$$

where

$$M(\alpha) = Y(c,\alpha)J(a,\alpha) - J(c,\alpha)Y(a,\alpha) \tag{39}$$

If we eliminate $\Phi_1^+(a,\alpha)$ of (30) and (38) equations,we get second Modified Wiener-Hopf equation as :

$$\frac{a}{2}V_3(\alpha)F_-(a,\alpha) - V_2(\alpha)e^{i\alpha l}\Phi_2^+(b,\alpha) + \frac{b}{2}G_1(b,\alpha)$$

$$= V_3(\alpha)\sum_{m=1}^{\infty}\frac{J(b,\alpha_m)}{\pi J(a,\alpha_m)}\frac{(ik\eta + i\alpha)f_m}{\alpha_m^2 - \alpha^2} - \sum_{m=1}^{\infty}\frac{(ik\eta + i\alpha)f_m}{\pi(\alpha_m^2 - \alpha^2)}$$

$$+e^{i\alpha l}\sum_{m=1}^{\infty}\frac{J(c,\beta_m)}{\pi J(b,\beta_m)}\frac{(ik\eta + i\alpha)g_m}{\beta_m^2 - \alpha^2} - \frac{a}{2i}\frac{V_3(\alpha)A_nJ_0(\gamma_n)}{(\lambda_n + \alpha)} \tag{40}$$

here

$$V_2(\alpha) = \frac{J(c,\alpha)}{\pi J(b,\alpha)N(\alpha)} \qquad \textit{and} \qquad V_3(\alpha) = \frac{J(a,\alpha)}{J(b,\alpha)} \tag{41}$$

Performing standard factorization and decomposition procedures and then applying Liouville's theorem we get

$$\frac{a}{2}\frac{F_-(a,\alpha)}{V_1^-(\alpha)} = \sum_{m=1}^{\infty}\frac{e^{i\delta_m l}J(a,\delta_m)V_1^+(\delta_m)\Phi_2^+(b,\delta_m)}{J'(b,\delta_m)(\delta_m - \alpha)} - \sum_{m=1}^{\infty}\frac{J(b,\alpha_m)}{\pi V_1^+(\alpha_m)J(a,\alpha_m)}\frac{(ik\eta - i\alpha_m)f_m}{2\alpha_m(\alpha_m + \alpha)}$$

$$+ \sum_{m=1}^{\infty}\frac{J(b,\alpha_m)}{\pi V_1^-(\alpha)J(a,\alpha_m)}\frac{(ik\eta + i\alpha)f_m}{\alpha_m^2 - \alpha^2} - \frac{a}{2i}\frac{A_nJ_0(\gamma_n)}{V_1^-(\alpha)(\lambda_n + \alpha)} + \frac{a}{2i}\frac{A_nJ_0(\gamma_n)}{V_1^+(\lambda_n)(\lambda_n + \alpha)} \tag{42a}$$

$$V_1^+(\alpha)\Phi_1^+(a,\alpha) = \frac{e^{i\alpha l}J(a,\alpha)V_1^+(\alpha)\Phi_2^+(b,\alpha)}{J(b,\alpha)} + \sum_{m=1}^{\infty}\frac{e^{i\delta_m l}J(a,\delta_m)V_1^+(\delta_m)\Phi_2^+(b,\delta_m)}{J'(b,\delta_m)(\delta_m-\alpha)}$$

$$-\sum_{m=1}^{\infty}\frac{J(b,\alpha_m)}{\pi V_1^+(\alpha_m)J(a,\alpha_m)}\frac{(ik\eta - i\alpha_m)f_m}{2\alpha_m(\alpha_m+\alpha)} + \frac{a}{2i}\frac{A_nJ_0(\gamma_n)}{V_1^+(\lambda_n)(\lambda_n+\alpha)} \quad (42b)$$

and

$$V_2^+(\alpha)\Phi_2^+(b,\alpha) = \frac{a}{2}\sum_{m=1}^{\infty}\frac{J(a,\delta_m)e^{i\delta_m l}}{J'(b,-\delta_m)}\frac{F_-(a,-\delta_m)}{V_2^+(\delta_m)(\delta_m+\alpha)}$$

$$-\sum_{m=1}^{\infty}\sum_{p=1}^{\infty}\frac{J(b,\alpha_m)J(a,\delta_p)(ik\eta - i\delta_p)e^{i\delta_p l}f_m}{\pi J(a,\alpha_m)J'(b,-\delta_p)V_2^+(\delta_p)(\alpha_m^2-{\delta_p}^2)(\delta_p+\alpha)}$$

$$-\sum_{m=1}^{\infty}\frac{J(c,\beta_m)(ik\eta - i\beta_m)g_m}{\pi V_2^+(\beta_m)J(b,\beta_m)2\beta_m(\beta_m+\alpha)} + \frac{a}{2i}\sum_{m=1}^{\infty}\frac{J(a,\delta_m)e^{i\delta_m l}A_nJ_0(\gamma_n)}{J'(b,-\delta_m)V_2^+(\delta_m)(\lambda_n-\delta_m)(\delta_m+\alpha)} \quad (42c)$$

We get the following solution from the common solution of the equations :

$$\Phi_1^+(a,\alpha) = \frac{e^{i\alpha l}V_3(\alpha)}{V_2^+(\alpha)}\{\frac{a}{2}\sum_{m=1}^{\infty}\frac{J(a,\delta_m)e^{i\delta_m l}}{J'(b,-\delta_m)}\frac{F_-(a,-\delta_m)}{V_2^+(\delta_m)(\delta_m+\alpha)}$$

$$-\sum_{m=1}^{\infty}\sum_{p=1}^{\infty}\frac{J(b,\alpha_m)J(a,\delta_p)(ik\eta - i\delta_p)e^{i\delta_p l}f_m}{\pi J(a,\alpha_m)J'(b,-\delta_p)V_2^+(\delta_p)(\alpha_m^2-{\delta_p}^2)(\delta_p+\alpha)} - \sum_{m=1}^{\infty}\frac{J(c,\beta_m)(ik\eta - i\beta_m)g_m}{\pi V_2^+(\beta_m)J(b,\beta_m)2\beta_m(\beta_m+\alpha)}$$

$$+\frac{a}{2i}\sum_{m=1}^{\infty}\frac{J(a,\delta_m)e^{i\delta_m l}A_nJ_0(\gamma_n)}{J'(b,-\delta_m)V_2^+(\delta_m)(\lambda_n-\delta_m)(\delta_m+\alpha)}\} + \frac{1}{V_1^+(\alpha)}\sum_{m=1}^{\infty}\frac{e^{i\delta_m l}J(a,\delta_m)V_1^+(\delta_m)\Phi_2^+(b,\delta_m)}{J'(b,\delta_m)(\delta_m-\alpha)}$$

$$-\frac{1}{V_1^+(\alpha)}\sum_{m=1}^{\infty}\frac{J(b,\alpha_m)}{\pi V_1^+(\alpha_m)J(a,\alpha_m)}\frac{(ik\eta - i\alpha_m)f_m}{2\alpha_m(\alpha_m+\alpha)} + \frac{a}{2i}\frac{A_nJ_0(\gamma_n)}{V_1^+(\lambda_n)V_1^+(\alpha)(\lambda_n+\alpha)} \quad (43)$$

2.3 Determination of the expansion coefficients

The expression of $\mathbf{\Phi}_1^+(a,\alpha)$ in (43) involves the unknown constant $f_m, g_m, \mathbf{\Phi}_2^+(b,\delta_m)$ and $F_-(a,-\delta_m)$. To determine these constants we substitute $\alpha = \alpha_1, \alpha_2, \cdots \alpha_N$ in (42b) , $\alpha = -\delta_1, -\delta_2, \cdots -\delta_N$ in (42a), $\alpha = \beta_1, \beta_2, \cdots \beta_N$ in (43c) and $\alpha = \delta_1, \delta_2, \cdots \delta_N$ in (43c) and use correlation of f_m *and* g_m coffecients with $\mathbf{\Phi}_1^+(a,\alpha_m)$ *and* $\mathbf{\Phi}_2^+(b,\beta_m)$, then we get following infinite systems of linear algebraic equations:

$$-\frac{J(a,\alpha_r)}{\pi J(b,\alpha_r)}\vartheta_r^1(ik\eta+i\alpha_r)V_1^+(\alpha_r)f_r=\sum_{m=1}^{\infty}\frac{e^{i\delta_m l}J(a,\delta_m)V_1^+(\delta_m)\Phi_2^+(b,\delta_m)}{J'(b,\delta_m)(\delta_m-\alpha_r)}$$
$$-\sum_{m=1}^{\infty}\frac{J(b,\alpha_m)}{\pi V_1^+(\alpha_m)J(a,\alpha_m)}\frac{(ik\eta-i\alpha_m)f_m}{2\alpha_m(\alpha_m+\alpha_r)}+\frac{a}{2i}\frac{A_nJ_0(\gamma_n)}{V_1^+(\lambda_n)(\lambda_n+\alpha_r)}\quad,\ r=1,2,\cdots\quad(44a)$$

$$-\frac{J(b,\beta_r)}{\pi J(c,\beta_r)}\vartheta_r^2(ik\eta+i\beta_r)V_2^+(\beta_r)h_r=\frac{a}{2}\sum_{m=1}^{\infty}\frac{J(a,\delta_m)e^{i\delta_m l}}{J'(b,-\delta_m)}\frac{F_-(a,-\delta_m)}{V_2^+(\delta_m)(\delta_m+\beta_r)}-$$
$$\sum_{m=1}^{\infty}\sum_{p=1}^{\infty}\frac{J(b,\alpha_m)J(a,\delta_p)(ik\eta-i\delta_p)e^{i\delta_p l}f_m}{\pi J(a,\alpha_m)J'(b,-\delta_p)V_2^+(\delta_p)(\alpha_m^2-\delta_p{}^2)(\delta_p+\beta_r)}-\sum_{m=1}^{\infty}\frac{J(c,\beta_m)(ik\eta-i\beta_m)h_m}{\pi V_2^+(\beta_m)J(b,\beta_m)2\beta_m(\beta_m+\beta_r)}$$
$$+\frac{a}{2i}\sum_{m=1}^{\infty}\frac{J(a,\delta_m)e^{i\delta_m l}A_nJ_0(\gamma_n)}{J'(b,-\delta_m)V_2^+(\delta_m)(\lambda_n-\delta_m)(\delta_m+\beta_r)}\quad,\quad r=1,2,\cdots\quad(44b)$$

$$\frac{a}{2}\frac{F_-(a,-\delta_r)}{V_1^+(\delta_r)}=\sum_{m=1}^{\infty}\frac{e^{i\delta_m l}J(a,\delta_m)V_1^+(\delta_m)\Phi_2^+(b,\delta_m)}{J'(b,\delta_m)(\delta_m-\delta_r)}-\sum_{m=1}^{\infty}\frac{J(b,\alpha_m)}{\pi V_1^+(\alpha_m)J(a,\alpha_m)}\frac{(ik\eta-i\alpha_m)f_m}{2\alpha_m(\alpha_m-\delta_r)}$$
$$+\sum_{m=1}^{\infty}\frac{J(b,\alpha_m)}{\pi V_1^+(\delta_r)J(a,\alpha_m)}\frac{(ik\eta-i\delta_r)f_m}{\alpha_m^2-\delta_r{}^2}-\frac{a}{2i}\frac{A_nJ_0(\gamma_n)}{V_1^+(\delta_r)(\lambda_n-\delta_r)}$$
$$+\frac{a}{2i}\frac{A_nJ_0(\gamma_n)}{V_1^+(\lambda_n)(\lambda_n-\delta_r)}\quad,\quad r=1,2,\cdots\quad(44c)$$

$$V_2^+(\delta_r)\Phi_2^+(b,\delta_r)=\frac{a}{2}\sum_{m=1}^{\infty}\frac{J(a,\delta_m)e^{i\delta_m l}}{J'(b,-\delta_m)}\frac{F_-(a,-\delta_m)}{V_2^+(\delta_m)(\delta_m+\delta_r)}-$$
$$\sum_{m=1}^{\infty}\sum_{p=1}^{\infty}\frac{J(b,\alpha_m)J(a,\delta_p)(ik\eta-i\delta_p)e^{i\delta_p l}f_m}{\pi J(a,\alpha_m)J'(b,-\delta_p)V_2^+(\delta_p)(\alpha_m^2-\delta_p{}^2)(\delta_p+\delta_r)}-\sum_{m=1}^{\infty}\frac{J(c,\beta_m)(ik\eta-i\beta_m)h_m}{\pi V_2^+(\beta_m)J(b,\beta_m)2\beta_m(\beta_m+\delta_r)}$$
$$+\frac{a}{2i}\sum_{m=1}^{\infty}\frac{J(a,\delta_m)e^{i\delta_m l}A_nJ_0(\gamma_n)}{J'(b,-\delta_m)V_2^+(\delta_m)(\lambda_n-\delta_m)(\delta_m+\delta_r)}\quad,\quad r=1,2,\cdots\quad(44d)$$

2.4 Analysis of the Field

The transmitted field in the region $\rho < a\,, z > 0$ can be obtained by taking inverse Fourier transform of $F_+(\rho,\alpha)$. From (18) and (19) we obtain

$$u_1(\rho,z)=\frac{1}{2\pi}\int_{\mathcal{L}}\left[\frac{J_0(K\rho)}{J(a,\alpha)}\Phi_1^+(a,\alpha)-F^+(\rho,\alpha)\right]e^{-i\alpha z}d\alpha \quad (45)$$

where $\mathcal{L}$ is a straight line parallel to the real $\alpha -$ axis, lying in the strip $Im(-k) < Im(\alpha) < Im(k)$. By using the theorem of residues we obtain easily.

$$u_1(\rho, z) = -i\sum_{n=1}^{\infty}\frac{J_0(\gamma_n\rho/a)}{J'(a,-\lambda_n)}\Phi_1^+(a,-\lambda_n)e^{i\lambda_n z} \tag{46}$$

where $\Phi_1^+(a,\alpha)$ is given by (43) and

$$J'(a,-\lambda_n) = \frac{-\lambda_n a}{\gamma_n}\,[ika\eta J_1(\gamma_n) - \gamma_n J_0(\gamma_n)]\ ,\ \ \lambda_n = \sqrt{k^2 - (\gamma_n/a)^2}\quad ,\quad n = 1,2,3,\ldots \quad (47a,b)$$

the transmission and similarly refflection coefficient is

$$T = -i\frac{\Phi_1^+(a,-\lambda_1)}{J'(a,-\lambda_1)}\ ,\quad R = i\frac{\Phi_1^+(a,\lambda_1)}{J'(a,\lambda_1)} \qquad (48,49)$$

3. Conclusions

Scattering of sound waves in a two-stepped cylindrical duct whose walls are coated by a sound absorbing lining is investigated by Wiener-Hopf technique. By using fourier transform technique it is obtained that a pair of modified Wiener-Hopf equations whose solution consists of four sets of infinitely many unknown expansion coefficients providing four systems of linear algebraic equations.

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